What needs to be in your head for Exam 2

Oscillations

Require restoring force, $F_{Restore}$ = -kx that opposes displacement

Restoring force (required for all oscillations): $\vec{F}_{Restore} = -k\vec{x}$

negative sign ⇒ Force opposes displacement from equilibrium

Damping force (drag, not surface friction): $\vec{F}_{Damping} = -b\dot{\vec{x}}$

negative sign ⇒ Force opposes velocity

Driving force (assumed harmonic): $\vec{F}_{Driving} = \vec{F}_0 \cos\left(\omega_{Drive} t\right)$

Simple Oscillations: Simple Harmonic Motion (SHM), restoring force only.

Apply NSL: $m\ddot{x} = -kx$

Rearrange to write DE: $\ddot{x} + \omega_N^2 x = 0$, where $\omega_N = \sqrt{\frac{k}{m}}$ is the natural frequency

Solution is some form of $x(t) = ACos(\omega_N t - \delta)$

Frequencies: $\omega = 2\pi f$ ~ radians/second, f ~ s^{-1} ~ Hertz, Hz,

Period $\tau = 1/f = 2\pi/\omega$ ~ seconds

Damped Oscillations, restoring force and damping (drag) force.

Damping force adds to restoring force, $F_{Damping} = -b\dot{x}$, b = damping coefficient

Apply NSL: $m\ddot{x} = -kx - b\dot{x}$

Rearrange to write DE: $\ddot{x} + 2\beta\dot{x} + \omega_N^2 x = 0$, where $\beta = \frac{b}{2m}$ is the damping parameter

Solutions depend on value of β vs. ω_N

$\omega_N > \beta$ ⇒ Under damped: $x = A_0 e^{-\beta t} Cos\left(\omega_S t - \delta\right)$, $\omega_S = \sqrt{\omega_N^2 - \beta^2}$, $\tau_S = 2\pi/\omega_S$

Decrement of motion (to find β) compare amplitudes at t_0 and later time Δt or $n\tau_S$

$$\frac{A\left(t_0\right)}{A\left(t_0 + \Delta t\right)} = \frac{A_0 e^{-\beta t_0}}{A_0 e^{-\beta(t_0+\Delta t)}} = e^{\beta\Delta t} \quad \text{or} \quad \frac{A_0 e^{-\beta t_0}}{A_0 e^{-\beta(t_0+n\tau)}} = e^{\beta n\tau}$$

$\omega_N = \beta$ ⇒ Critically damped

$\omega_N < \beta$ ⇒ Over damped

Driven Damped Oscillations, restoring force, damping, and driving forces

Harmonic driving force: $F_{Driving} = +F_0 Cos(\omega_{Driving} t)$, F_0 = amplitude of applied force

Apply NSL: $m\ddot{x} = -kx - b\dot{x} + F_0\cos\left(\omega_D t\right)$

Rearrange to write DE: $\ddot{x} + 2\beta\dot{x} + \omega_N^2 x = F_0\cos\left(\omega_D t\right)$

Solution is a linear sum of solutions to the homogeneous & inhomogeneous DEs

$x(t) = x_{complimentaory}(t) + x_{particular}(t)$

$x_c(t)$ is the solution to the homogeneous DE for ω_N^2 vs. β^2 above

$x_p(t)$ = is a solution to the inhomogeneous equation